Kinks of arbitrary width

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The influence of external potentials on kink width is investigated. The stability of a single kink and kink-antikink solutions in the presence of an external force is proved. The consequences of stability for defect production during a quench are discussed.

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I. INTRODUCTION

Recently much attention has been focused on the formation of topological defects in the condensed matter systems. Continuous phase transitions are still the most interesting in this context. The mechanism of the creation of topological solutions during the phase transition was first described by Kibble and Zurek [1]. They noticed that as a consequence of critical slowing down the relaxation time diverges and perturbations of the order parameter propagate very slowly. If the time of the propagation of density perturbations over the correlated regions becomes comparable with the relaxation time, the field configuration in the system freezes-in. Immediately after transition, the system regains the capacity to respond to the change of external parameters. The correlation length at that instant (freeze-out instant) sets the size of the region over which the same vacuum can be selected. Hence, it sets the resulting density of the topological defects. The correlation length at that instant describes the size of the defect and therefore the density of defects is limited by their size at freeze-out time. This scenario was confirmed in numerical experiments [2].

This description mainly concerns pure systems driven by the temperature noise. On the other hand, we know that a quite reasonable class of the systems is inevitably populated by the impurities and admixtures. The most representative examples are liquid crystals and superconducting layers. The superconductors of the second type seem to be particularly useful in testing the influence of impurities on defect formation. On the other hand, the transitions in liquid crystals can be only approximately described using the Kibble-Zurek scenario. The main reason lies in the existence of a small energy barrier near the critical temperature that makes these transitions belong to the so-called weak first-order transitions. The considerations of the influence of impurities on defect production were performed in Ref. [3]. It was proved that the number density of produced defects can be determined not only by the correlation length, but also by the characteristic length scales of the impurity distributions. These results seem to be quite natural in the case of smooth, strong and long range impurity potentials. The long range of the external potential means that the mean distance between the zeros of the order parameter generated by the impurity potential is significantly larger than the correlation length itself.

The present paper aims at working out a intuition concerning the opposite regime. It is known that the number density of produced kinks is limited by their size. In the following sections, using examples of exact solutions, it is shown that the external forces coming from the impurities can squeeze the kink to an arbitrary size, leaving more room for the production of other kinks in its vicinity.

The paper is organized as follows. In the following section, an exact squeezed kink solution in the presence of some external force distribution is found. Section III contains the stability analysis of the squeezed kink in the overdamped Landau-Ginzburg model. The influence of the inertia force on the stability of this solution is discussed in Sec. IV. The late stage of the evolution of the kink network produced during the transition in the presence of the external potential is illustrated, using example of an exact kink-antikink solution, in Sec. V. The stability of this solution is investigated in Sec. VI. The concluding section contains remarks.

II. KINK SOLUTION OF AN ARBITRARY WIDTH

Let us consider an overdamped Landau-Ginzburg model in one spatial dimension,

$$\Gamma \partial_t \phi(t,x) = \partial_x^2 \phi(t,x) + a\phi(t,x) - \lambda \phi^3(t,x) + \mathcal{D}(x), \quad (1)$$

where the quantity $\mathcal{D}(t,x)$ represents a deterministic force describing the existence of impurities or crystalline net in the substance. The static squeezed kinks are solutions of the ordinary nonlinear inhomogeneous equation

$$-\partial_x^2 \phi(x) - a\phi(x) + \lambda \phi^3(x) = \mathcal{D}(x). \quad (2)$$

Let us choose the particular form of the force distribution

$$\mathcal{D}(x) = \pm \mathcal{A} \left(\frac{a}{\lambda} \right)^{3/2} \frac{\sinh \beta(x - x_0)}{\cosh^3 \beta(x - x_0)}, \quad (3)$$

where $\mathcal{A} \in [0,\infty)$ describes the strength of the impurity force, $\beta \equiv \sqrt{(a/2)}\gamma$ and $\gamma \equiv \sqrt{1 + \mathcal{A}/\lambda}$. The kink solution of Eq. (2) can be found with the help of the standard procedure [4]. First, the order of this equation can be lowered by one,

$$[\partial_x \phi(x)]^2 = \frac{\lambda}{2} \left(\phi^2(x) - \frac{a}{\lambda} \right)^2 - \mathcal{V}(x), \quad (4)$$

where $\partial_x \mathcal{V}(x) \equiv 2\mathcal{D}(x)\partial_x \phi(x)$. Next integration (depending on the sign of the force distribution) leads to the squeezed kink

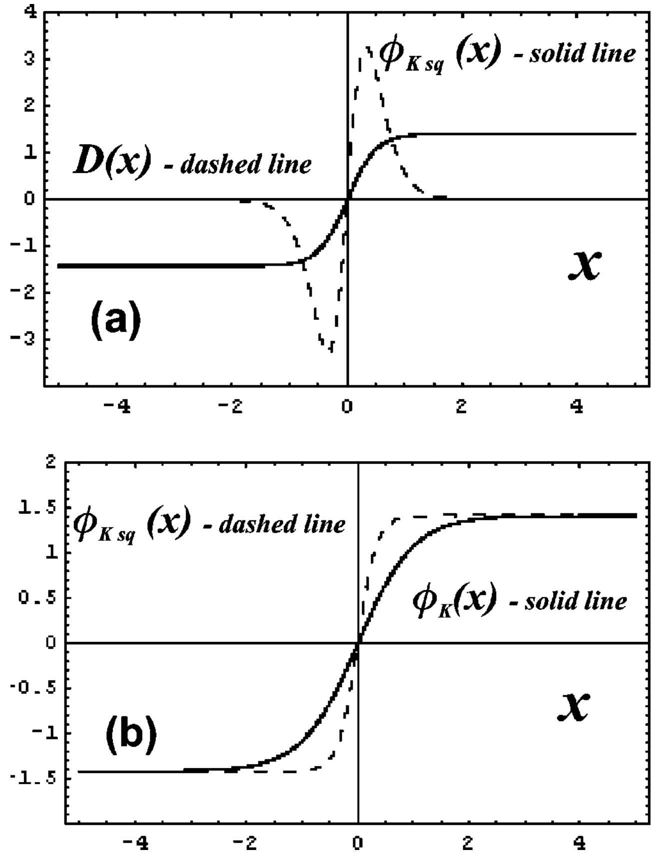

FIG. 1. (a) The squeezed kink solution (solid line) and the impurity force distribution (dashed line). Parameters chosen in this plot are the following: $a=2$, $\lambda=1$, $\mathcal{A}=3$, $x_0=0$, $\gamma=2$. (b) The kink solution in the absence of the external force (solid line) and the squeezed kink solution (dashed line). In the case of the squeezed kink the amplitude of the external force is the following: $\mathcal{A}=8$ and therefore $\gamma=3$.

$$\phi_{K\,sq}(x-x_0)=\sqrt{\frac{a}{\lambda}}\tanh\left(\sqrt{\frac{a}{2}}\gamma(x-x_0)\right), \tag{5}$$

or squeezed antikink solution

$$\phi_{A\,sq}(x-x_0)=-\sqrt{\frac{a}{\lambda}}\tanh\left(\sqrt{\frac{a}{2}}\gamma(x-x_0)\right). \tag{6}$$

Because the field strength parameter $\mathcal{A}$ is larger or equal to zero $\mathcal{A}\in[0,\infty)$, the squeezing parameter γ belongs to the interval $\gamma\in[1,\infty)$. In the absence of impurities $\mathcal{A}=0$, and therefore the kink is unsqueezed, i.e., $\gamma=1$ (see Fig. 1).

III. STABILITY OF THE KINK IN AN OVERDAMPED ϕ^4 MODEL

Let us consider time dependent solutions of the overdamped ϕ^4 model,

$$\Gamma\partial_t\phi(t,x)=\partial_x^2\phi(t,x)+a\phi(t,x)-\lambda\phi^3(t,x)+\mathcal{D}(x). \tag{7}$$

We will focus on the small, time dependent, perturbations of the kink solution $\phi(t,x)=\phi_{K\,sq}(x)+\xi(t,x)$. For simplicity, the kink under consideration is located at the zero of the coordinate system $x_0=0$. We will consider the perturbations of the form $\xi(t,x)=e^{-(\Omega/\Gamma)t}u(x)$, where a function $u(x)$ vanishes at spatial infinity. In the linear approximation, the equation of motion reduces to the eigenvalue problem

$$\tilde{\Omega}u(z)=\partial_z^2u(z)+2[1-3\tanh^2(\gamma z)]u(z), \tag{8}$$

where we introduced a new variable $z\equiv\sqrt{(a/2)}x$ and rescaled constant $\tilde{\Omega}\equiv(2/a)\Omega$. We find the complete spectrum of the kink excitations. The ground state is the following:

$$u_0(x)=\left(\gamma\frac{\Gamma\left(\alpha+\frac{1}{2}\right)}{\Gamma(\alpha)}\sqrt{\frac{a}{2\pi}}\right)^{1/2}\frac{1}{\cosh^{\alpha}\left(\sqrt{\frac{a}{2}}\gamma x\right)}, \tag{9}$$

where

$$\alpha\equiv\frac{1}{2}\left(\sqrt{1+\frac{24}{\gamma^2}}-1\right).$$

This solution exists for an arbitrary thin potential "hole," i.e., for $\gamma\in[1,\infty)$. The notation refers to the fact that in the absence of the external potential it corresponds to the zero mode [5]. The eigenvalue $\Omega_0=a/2(\gamma^2\alpha-2)$ is equal to zero only for $\gamma=1$. We know that the zero mode is a manifestation of the translational symmetry of the system. In fact $\Omega_0=0$ only if the system is not occupied by the impurities, i.e., for $\mathcal{A}=0$. For $\mathcal{A}>0$, the "energy" of this mode is positive $\Omega>0$. The next eigenfunction is a breather mode,

$$u_3(x)=\left(\gamma(2\alpha-1)\frac{\Gamma\left(\alpha-\frac{1}{2}\right)}{\Gamma(\alpha-1)}\sqrt{\frac{a}{2\pi}}\right)^{1/2}\times\frac{\sinh\left(\sqrt{\frac{a}{2}}\gamma x\right)}{\cosh^{\alpha}\left(\sqrt{\frac{a}{2}}\gamma x\right)}. \tag{10}$$

This mode corresponds to the eigenvalue $\Omega_3=a/2[\gamma^2(3\alpha-1)-2]$. The breather mode is separated by a gap from the ground state. This mode exists in the system only for the parameter range $\gamma\in[1,\sqrt{3})$. Due to the fact that γ describes the thickness of the potential distribution it is obvious that this mode exists only for a sufficiently wide potential hole. Finally, we also obtain the continuous spectrum of the eigenfunctions

$$u_k(x)=\left(\frac{\gamma}{2\pi}\sqrt{\frac{a}{2}}\right)^{1/2}e^{ik\sqrt{(a/2)}\gamma x}{}_2F_1\times\left(-\alpha,\alpha+1;1-ik;\frac{1-\tanh\left(\sqrt{\frac{a}{2}}\gamma x\right)}{2}\right),\tag{11}$$

which corresponds to the eigenvalues $\Omega_k=a/2(4+\gamma^2k^2)$. The modes of this type exists for arbitrary values of the parameter $\gamma\in[1,\infty)$. The bound states are normalized to unity and the states that correspond to the continuum spectrum are normalized so that

$$\int_{-\infty}^{\infty}dx u_k^*(x)u_{k'}(x)=\delta(k-k').\tag{12}$$

In the particular example of $a=2$ and considering the lack of the external force $\gamma=1$, we recover (up to normalization factor) the result of Ref. [5],

$$u_k(x)=\frac{1}{\sqrt{2\pi}}\frac{1+k^2}{(k+i)(k+2i)}e^{ikx}\left[1+\frac{3ik\tanh x-3\tanh^2 x}{1+k^2}\right].$$

We used the fact that for negative integers the expansion of the hypergeometric function ${}_2F_1(-2,3;1-ik;[1-\tanh(x)]/2)$ consists of a finite number of terms.

IV. INFLUENCE OF THE INERTIA FORCE ON KINK STABILITY

Let us also check the stability of the squeezed kink solution in the case of the equation of motion equipped with a term with the second time derivative of the order parameter

$$m\partial_t^2\phi(t,x)+\Gamma\partial_t\phi(t,x)=\partial_x^2\phi(t,x)+a\phi(t,x)-\lambda\phi^3(t,x)+\mathcal{D}(x).\tag{13}$$

Similarly, as in the overdamped model, we consider small perturbations of the kink $\phi(t,x)=\phi_{K\,sq}(x)+\xi(t,x)$ located at the zero position of the x coordinate, i.e., $x_0=0$. We adopt perturbations of the form $\xi(t,x)=e^{-(\omega/\Gamma)t}u(x)$. The number of modes in this model is twice as much as in the overdamped model. The modes in this model can be easily found. One of the lowest exponents,

$$\omega_0^{\pm}=\frac{\Gamma^2}{2m}\left(1\pm\sqrt{1-\frac{2am}{\Gamma^2}(\gamma^2\alpha-2)}\right),\tag{14}$$

corresponds to the zero mode of the overdamped model. The excitations of the kink are stable if $\mathrm{Re}\,\omega_0^{\pm}\geqslant 0$. The zero mode $\mathrm{Re}\,\omega_0^{\pm}=0$ is present only in the case where the external force $\mathcal{A}=0\Leftrightarrow\gamma=1$ is absent. In this case, translational invariance of the model is restored. If, however, the external force is present, then $\gamma>1$ and for stability of the "zero" modes we need $\mathrm{Re}[\sqrt{1-2am(\gamma^2\alpha-2)/\Gamma^2}]<1$. This inequality is satisfied if $\gamma^2\alpha-2>0$. It is easy to check that the last inequality is fulfilled always for $\gamma>1$. Let us also notice that if $\mathrm{Im}[\sqrt{1-2am(\gamma^2\alpha-2)/\Gamma^2}]\neq 0$, then the excitations oscillate with the frequency $\Gamma^2\sqrt{2am(\gamma^2\alpha-2)/\Gamma^2-1}/2m$. The oscillations of "zero" modes always appear if $(\gamma^2\alpha-2)>\Gamma^2/2am$. The breather modes correspond to the exponents

$$\omega_3^{\pm}=\frac{\Gamma^2}{2m}\left(1\pm\sqrt{1-\frac{2am}{\Gamma^2}[\gamma^2(3\alpha-1)-2]}\right).\tag{15}$$

The modes of this type are stable if $\gamma^2(3\alpha-1)-2>0$, which is always satisfied for $\gamma\in[1,\sqrt{3})$. The modes that belong to the continuous spectrum

$$\omega_k^{\pm}=\frac{\Gamma^2}{2m}\left(1\pm\sqrt{1-\frac{2am}{\Gamma^2}(4+\gamma^2k^2)}\right)\tag{16}$$

are also stable, because the inequality $4+\gamma^2k^2>0$ is always satisfied. Oscillations in these cases are also possible.

V. THE KINK-ANTIKINK STATIC SOLUTION

After the phase transitions in the pure medium the kink-antikink pairs annihilate, leaving the system free of any defects. The situation in the medium populated by the impurities can be completely different. In the present section, the exact kink-antikink solution is constructed. This simple example shows that at the final stage of the phase transition the kinks and antikinks can form a stable network. All the kinks and antikinks belonging to this network are trapped by the impurities. Let us consider a time independent equation of motion,

$$-\partial_x^2\phi(x)-a\phi(x)+\lambda\phi^3(x)=\mathcal{D}(x).\tag{17}$$

We also choose the particular form of the impurity force distribution:

$$\mathcal{D}(x)=\mathcal{D}_+(x)+\mathcal{D}_-(x)+\mathcal{D}_I(x).\tag{18}$$

The first part of this distribution responsible for squeezing the vortex at the position x_0,

$$\mathcal{D}_+(x)=\mathcal{A}\left(\frac{a}{\lambda}\right)^{3/2}\frac{\sinh\beta(x-x_0)}{\cosh^3\beta(x-x_0)}.\tag{19}$$

The second part squeezes antikink at the position $-x_0$,

$$\mathcal{D}_-(x)=-\mathcal{A}\left(\frac{a}{\lambda}\right)^{3/2}\frac{\sinh\beta(x+x_0)}{\cosh^3\beta(x+x_0)}.\tag{20}$$

The last part of the force distribution is responsible for balancing the kink-antikink interaction,

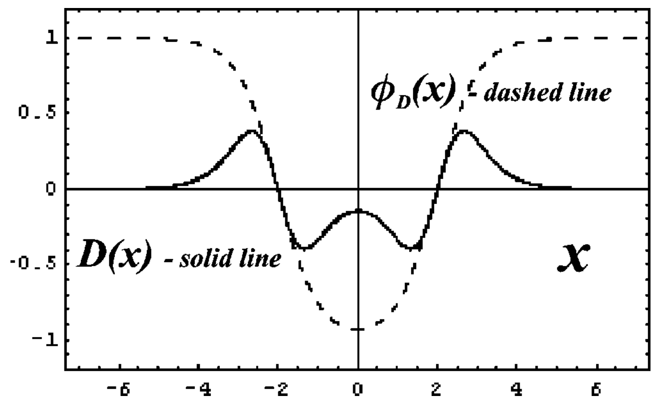

FIG. 2. The kink-antikink solution (dashed line) and the shape of the force distribution (solid line). Parameters chosen in this plot are the following: $a=1$, $\lambda=1$, $x_0=2$, $\mathcal{A}=1$.

$$\mathcal{D}_I(x)=3\lambda\left(\frac{a}{\lambda}\right)^{3/2}[\tanh\beta(x-x_0)-\tanh\beta(x+x_0)]\times[1+\tanh\beta(x-x_0)-\tanh\beta(x+x_0)-\tanh\beta(x-x_0)\tanh\beta(x+x_0)]. \quad (21)$$

In this setting the exact solution of the equation of motion (17) is the following:

$$\phi_D(x)=\sqrt{\frac{a}{2}}\left[\tanh\left(\sqrt{\frac{a}{2}}\gamma(x-x_0)\right)-\tanh\left(\sqrt{\frac{a}{2}}\gamma(x+x_0)\right)+1\right]. \quad (22)$$

This solution is just a superposition of the kink and antikink located at the zeros of the force distribution

$$\phi_D(x)=\phi_{K\,sq}(x-x_0)+\phi_{A\,sq}(x+x_0)+\sqrt{\frac{a}{2}}. \quad (23)$$

This solution represents the kink and antikink trapped by the neighboring impurity centers (see Fig. 2).

VI. STABILITY OF THE KINK-ANTIKINK SOLUTION

We know that the squeezed kink as well as the squeezed antikink are stable against small perturbations. The only instability of the kink-antikink solution can be introduced by a change in their relative position. Because of the symmetry of the force distribution, we consider the following perturbation of the kink-antikink solution:

$$\phi_D(t,x)\approx\phi_D(x)+\sqrt{\frac{a}{\lambda}}[F_K^2(x-x_0)+F_A^2(x+x_0)-2]\beta\epsilon(t), \quad (24)$$

where $\epsilon(t)$ is a small displacement of zeros from their position predicted by the exact solution (22). The approximate form of the kink-antikink solution follows from the expansion of the squeezed kink

$$\phi_{K\,sq}[x-x_0-\epsilon(t)]\approx\phi_{K\,sq}(x-x_0)+\sqrt{\frac{a}{2}}\beta[-1+F_K^2(x-x_0)]\epsilon(t), \quad (25)$$

and the squeezed antikink

$$\phi_{A\,sq}[x+x_0+\epsilon(t)]\approx\phi_{A\,sq}(x+x_0)+\sqrt{\frac{a}{2}}\beta[-1+F_A^2(x+x_0)]\epsilon(t), \quad (26)$$

where $\phi_{K\,sq}\equiv\sqrt{(a/2)}F_K$, $\phi_{A\,sq}\equiv\sqrt{(a/2)}F_A$. We consider the complete dynamics given by the equation of motion equipped with the inertia term

$$m\partial_t^2\phi_D(t,x)+\Gamma\partial_t\phi_D(t,x)=\partial_x^2\phi_D(t,x)+a\phi_D(t,x)-\lambda\phi_D^3(t,x)+\mathcal{D}(x). \quad (27)$$

First we perform expansion with respect to the small parameter $\beta\epsilon\ll 1$. Next, we use the reflection symmetry $x\leftrightarrow -x$, which allows us to restrict our considerations to perturbations of the position of the zero of the order parameter located in the neighborhood of the point x_0. At the final stage of the calculus, we use approximate values of the functions $F_K(0)\approx 0$ and $F_A(2x_0)\approx -1$ in the vicinity of x_0. The effective equation for the displacement of the positions of zeros of the scalar field is just the damped oscillator equation

$$m\partial_t^2\epsilon(t)+\Gamma\partial_t\epsilon(t)+(2\beta^2-a)\epsilon(t)=0. \quad (28)$$

The oscillations are stable if the parameter preceding ϵ is positive, i.e., if $\beta^2>a/2$. The last inequality can be written in the form $\gamma>1$, or equivalently $\mathcal{A}>0$. We see that if the configuration is placed in nonzero force then the excited kink-antikink solution relaxes to the static one. The way of this relaxation is described by the textbook solution

$$\epsilon(t)=Be^{-(\Gamma/2m)t}\cos(\varpi t+\varphi), \quad (29)$$

where the frequency of the oscillations is given by the potential parameter β as follows: $\varpi=\Gamma\sqrt{[4m(2\beta^2-a)/\Gamma^2]-1}/2m$. The main reason of the stabilization of the solution is an existence of the gradients of the potential in the vicinity of the positions of zeros of the order parameter.

VII. REMARKS

We considered simple examples of the exact solutions of ϕ^4 in the presence of external forces. The solutions illustrate the main features of the defect production in the systems populated by the impurities. We know that according to the Zurek scenario the density of defects produced during the phase transition of the second type is mainly determined by the correlation length at the freeze-out time. The correlation length at that instant of time intuitively describes the size of the defect, and therefore the number density of defects is limited by the possibility of holding the kinks in the unit volume. It was showed that the squeezed kink or antikink solutions can have a size much smaller than the kink solutions in the absence of the external forces. This seems to be a cause of the possible changes in the density of produced defects.

On the other hand, one could raise the question of whether the existence of the squeezed solutions is a generic feature of the model equipped in any external force distribution or is it only the unusual coincidence of the force distribution parameters? Let us assume that an arbitrary potential is described by some unknown function. The force distribution that is the first derivative of the potential disappears in extremes of the potential. On the other hand, in the vicinity of zeros of the force distribution nonzero gradients of the potential exists. These gradients corresponds to the forces acting on kinks located in considered areas. In these settings, considerable amount of the knots of the force distribution attract kinks. In a system equipped with a dissipation term, the movement of kinks, in the vicinity of the knot of the force distribution, is damped. The final state of this evolution is a static kinklike solution that differs from the squeezed kink by the local distortion of the profile of the squeezed kink. This distortion is a consequence of the difference between considered force distribution and the force distribution described in Sec. II.

The second important feature of the influence of the impurities on the evolution of the defect network is exemplified by the kink-antikink solution. We know that in pure systems, due to kink-antikink interactions at a sufficiently late instant of time, the defects disappear from the system completely. In the case of the system occupied by the impurities, due to the dissipation in the system, the surplus of the energy is lost and the system approaches the configuration that consists of defects oscillating around the impurity centers.

We know that in pure systems, due to the annihilation of defects and antidefects, the initial density of the defect network is quickly reduced in time. In fact, at this stage of evolution the nucleation is also possible but it is determined by the Boltzman factor [6]. In contradiction to pure systems, kinks produced in the systems populated by impurities are confined by the impurity centers and therefore may not disappear from the system completely.

It is worth emphasizing that the considerations of this paper concern the shape of possible field configurations of the order parameter in the broken symmetry phase of the ϕ^4 model and not the phase transition itself. Although the dynamics of the transition introduces some complications to this picture, the main features of the presented picture will surely survive even if the complete transition process will be taken in to account.

The next issue concerns the possible applicability of the obtained results to two- and three-dimensional systems. First, let us turn to the question whether the squeezed solutions exist in higher number of spatial dimensions. In d spatial dimensions $O(d)$ symmetric system is described by the equations of motion

$$\partial_t^2 \phi^a(t,\vec{x}) + \Gamma \partial_t \phi^a(t,\vec{x}) = \Delta \phi^a(t,\vec{x}) + a\phi^a(t,\vec{x}) - \lambda(\phi^b\phi^b)\phi^a(t,\vec{x}). \quad (30)$$

We assume that the number of real scalar fields in the model is identical to a number of spatial dimensions, i.e., $a = 1,2,\ldots,d$. This assumption guarantees existence of hedgehog solutions in the model. In two dimensions, we have the vortex solution $\phi_V^a = \phi_V^a(x)$, and in three spatial dimensions the global monopole solution exists $\phi_M^a = \phi_M^a(x)$. In the presence of the external force distribution $\mathcal{D}^a(t,\vec{x})$, this equation has the form

$$\partial_t^2 \phi^a(t,\vec{x}) + \Gamma \partial_t \phi^a(t,\vec{x}) = \Delta \phi^a(t,\vec{x}) + a\phi^a(t,\vec{x}) - \lambda(\phi^b\phi^b)\phi^a(t,\vec{x}) + \mathcal{D}^a(t,\vec{x}). \quad (31)$$

In two dimensions, the squeezed vortices $\phi_{V\,sq}^a(x) = \phi_V^a(\gamma x)$ exist for particular force distribution of the impurity $\mathcal{D}^a(x) = \mathcal{B}(a/\lambda - \phi_V^b\phi_V^b)\phi_V^a$. The explicit form of this distribution is not known because we also do not know the explicit form of the vortex solution. The squeezing factor this time is a combination of the constants characterizing the self-coupling of the scalar fields and the impurity strength $\gamma = \gamma(\mathcal{B},\lambda)$. Similarly, in the three-dimensional case, the squeezed monopoles $\phi_{M\,sq}^a(x) = \phi_M^a(\gamma x)$ appear for the force distribution $\mathcal{D}^a(x) = \mathcal{B}(a/\lambda - \phi_M^b\phi_M^b)\phi_M^a$. Moreover, the dynamics of these models is, as in one-dimensional systems, determined by gradients of the potential. In two and three dimensions we also can expect that gradients of an arbitrary potential could respond not only for confining, but also for local change of the profile of the vortex or monopole solutions. The dissipation in the system stabilizes the defects confined by the impurities. The number density of defects trapped by the impurities in this case is determined by the average separation of the impurity centers. The presence of the length scale of the impurity distribution in the number density formula of produced defects, in two and three spatial dimensions, can also be confirmed in the framework of the Kibble-Zurek scenario [7].

ACKNOWLEDGMENT

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